RELIABILITY ANALYSIS OF REINFORCED CONCRETE COLUMN UNDER CORROSION ATTACK

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Abstract

Corrosion of reinforcement is a major problem affecting a large number of reinforced concrete structures. At present, most reliability-based design studies of reinforced concrete structures do not consider the effects of corrosion.

In this paper, we present the reliability of reinforced concrete columns under corrosion attack. The used limit states equation in the analysis based on the Egyptian code and the analysis is performed by using Monte Carlo simulation technique. The effect of corrosion on the reliability of reinforced concrete column with different longitudinal steel ratio is investigated. Also the effect of different corrosion rates and initial time of corrosion on the reliability index is discussed.

Moreover the effect of eccentricity due to construction error on the corroded steel bar in reinforced concrete column is studied. The required time to inspect and repair for the reinforced concrete column is presented.

Keywords: reliability analysis; reinforced concrete column; corrosion, Monte-Carlo

Introduction

Most of the design studies in reinforced concrete (RC) literature assume that the durability of RC structures can be taken for granted. However, many RC structures are exposure during their lifetime to corrosion attack. In this paper, the reliability of the RC concrete column under corrosion attack is based on the early work of Khalil, A. B. et al. is developed, in residential building due to random variable live load applied on the building of six floors and four bays with each floor.

In this study the effect of increase concrete strength with time is taken into consideration.

The limit state equation is formulated based on Egyptian code. The reliability analysis is performed by using Monte-Carlo simulation technique.
Corrosion attack

Within the past 20 years, the deterioration of RC structures due to reinforcement corrosion has become an important problem.

The concrete is alkaline because it contains microscopic pores with high concentrations of soluble calcium, sodium and potassium oxides. These oxides form hydroxides derived from the reactions between mix water and Portland cement particle which are very alkaline.

According to Broomfield⁴, a strong acid has pH=1 (or less), a strong alkali has pH=14 (or more), a neutral solution has pH=7. Concrete has a pH of 12 to 13. Steel corrodes at pH 10 to 11.

The alkaline condition lead to a ‘passive’ layer forming on the steel surface. A passive layer is a dense, impenetrable film, which, if fully established and maintained, prevents further corrosion of the steel. The layer formed on steel in concrete is probably part metal oxide/ hydroxide and part mineral from the cement. A true passive layer is a very dense, thin layer of oxide that leads to a very slow rate of oxidation (corrosion).

Once the passive layer breaks down then areas of rust will start appearing on the steel surface. The chemical reactions are the same whether corrosion occurs by chloride attack or carbonation.

In this study the corrosion is seen as a three phase process, the first phase spans from the time of construction to the time of corrosion initiation. This phase is the diffusion of CO₂ to cause depassivation. The second phase follows until unacceptable levels of section loss have occurred, the third phase is occurred through the second phase as deterioration which is the begins of cracking and spalling of concrete cover.

Carbonation is reach to the steel bars as a result of a low cement content, high water cement ratio and poor curing of the concrete.

A carbonation front proceeds into the concrete roughly following the laws of diffusion. These are most easily defined by the statement that the rate is inversely proportional to the thickness.

\[
\frac{\text{Dx}}{\text{dt}} = \frac{\text{Do}}{x}
\]

where x is distance from concrete surface faced to environment, t is time and Do is the diffusion constant. The diffusion constant Do is determined by the concrete quality. At the carbonation front there is a sharp drop in alkalinity from pH11-13 down to less than pH 8.
In reliability analysis of concrete structures, many researches used different probabilistic models to describe initiation and corrosion rate of steel bars in concrete.

Mori and Ellingwood\textsuperscript{4} used the Poisson process with parameters $\nu(t)$ to describe the initiation of corrosion following carbonation. The mean Poisson is the parameter $\nu(t)$ which was expressed as follow:

$$
\nu(w) = \begin{cases} 
0 & \text{for } w < t^* \\
\nu & \text{for } w \geq t^*
\end{cases}
$$

where $t^*$ is a deterministic time and they consider it 10 years and $\nu$ is the mean initiation rate of corrosion which consider equal to 0.2 / year.

On the second phase, typical corrosion rates of steel in various environments have been reported in recent years. Because the corrosion rate changes with environment, no accurate data is available to predict the real corrosion rate.

Based on the average corrosion rates reported in Ting\textsuperscript{5} and Mori and Ellingwood\textsuperscript{6} and Dan M. Frangopol\textsuperscript{7} three corrosion rates of 64, 89, 114 $\mu$m/year can be used to cover most cases of corrosion rates which depends on different environment conditions.

These rates may suggest a mean (i.e., 89 $\mu$m/yr) and a standard deviation (i.e., 25 $\mu$m/yr).

For phase three, according to El-Said et al\textsuperscript{8}, the time, $t_s$, in year, between initiation of corrosion and spalling of concrete is calculated from the following equations.

$$
t_s = \frac{0.08C}{d.C_r}
$$

where $C$ is the concrete cover in mm, $d$ is the diameter of the steel bar and $C_r$ is the mean corrosion rate.

**Capacity Loss**

The capacity of the reinforced concrete column is dependent on the cross section dimensions (concrete and steel area) and material strength (concrete strength and steel yield strength).

In case of uniform corrosion as shown in Fig. 1, the total longitudinal reinforcement area can be expressed as a function of time $t$ as follow:

$$
A_s(t) = \begin{cases} 
n\pi D_b^2 / 4 & \text{for } t \leq T_i \\
n\pi \left[ D_b - 2C_r(t - T_i) \right]^2 / 4 & \text{for } t > T_i
\end{cases}
$$
where \( D_b \) is the diameter of the bar, \( n \) is number of bars, \( T_i \) is time of corrosion initiation, and \( C_r \) is the rate of corrosion. This equation takes into account the uniform corrosion propagation process from all sides.

As corrosion gradually progresses, the remaining capacity of the reinforced concrete column decreases. The strength of the RC column will be calculated by the following equation.

\[
R_u = \frac{0.67F_{cu}(t).b.t.(d_1 / 2) + (As(t).F_y).d_1}{(e + d_1 / 2)}
\]

where, \( b \) is the width of the column, \( t \) is the length of the column, \( F_y \) is the steel bar yield strength, \( F_{cu}(t) \) is the concrete compressive strength as a function of time and \( A_s(t) \) is the steel bar area as a function of time due to corrosion.

**The Limit State Equation**

The limit state equation is a function of the resistance; \( R \), of the structural element and of the load effect; \( S \), acting on it; \( R \) and \( S \) are random variables.

\[
Z = \frac{R}{S}
\]

(4)

since \( R \) and \( S \) are random variables the \( Z \) is also a random variable.

Knowing the probability density function of the resistance; \( R \), and load effect; \( S \), the probability density function of; \( Z \), can be obtained.

\[
P_f = P[R / S < 1.0]
\]

(5)

Knowing \( \ln 1.0 = 0.0 \), Equation (5) becomes

\[
P_f = P[\ln(R / S) < 0],
\]

\[
P_f = \int_0^1 f_z(z)dz = F_z(1.0)
\]

(6)

where \( f_z(z) \) is the probability density function of the random variable, \( Z \).

If the probability density function of the variable, \( z \), is a normal distribution, the probability of failure will be calculated by the following equation.

\[
P_f = P[Z < 1.0] = \phi\left[\frac{1.0 - \mu_z}{\sigma_z}\right]
\]

(7)

where \( \phi \) is the cumulative standard normal distribution function.
Safety can be measured in terms of a “safety index”; \( \beta \), which was defined by Cornell\(^9\) using the second-moment format as the number of standard deviations of the safety factor \( z \) by which its mean exceeds 1.0.

**Parametric Study**

There is different parameters affect on the reliability of the reinforced concrete column under corrosion attack. However the corrosion itself depends on some variables related to environment and weather conditions.

Two types of reinforced concrete column sections’ dimensions are taken into consideration. For the first one, the dimensions of length and width are equal to 500 x 500 mm with longitudinal steel ratio equal to 1% where the second section is design to carry the same load with section dimensions 400 x 400 mm with longitudinal steel ratio equal to 4%.

The steel bars before using may have rust on the other hand, the steel bars may take no adequate time from manufacture to pouring the concrete to have a rust. Therefore, in these two cases, the initial time of corrosion is different so different initial times (0, 10, 20 years) of corrosion are taken to study their effect on the reliability of columns.

The assumed statistical properties of the distributions of all variables, which are shown in Table 1, are based on several previous researches.

The corrosion rates 0.064, 0.089, 0.114 mm/year are taken into consideration. These rates may suggest a mean (i.e. 0.089 mm/year) and a standard deviation (0.025 mm/year) as Frangopol\(^7\).

**Effect of Percentage of Longitudinal Steel**

The corrosion rates 0.064, 0.089, 0.114 mm/year are taken into consideration. These rates may suggest a mean (i.e. 0.089 mm/year) and a standard deviation (0.025 mm/year).

Also, two percent of longitudinal steel bar in concrete is taken 1% and 4%, which represents a wide range of percentage of longitudinal steel bars which are used in column design in Egyptian code EC-89.

After formulating the limit state equation as discussed, the reliability index of column is calculated for the assume three corrosion rates and percentage of steel bars at different life time using 10,000 trials of Monte-Carlo simulation.

The effect of the two percentages of the steel bars on the reliability index at corrosion rate equal to 0.064 mm/year along the lifetime is shown in Fig. 2. From this figure, one can find that the reliability index of the reinforced concrete column is decreased gradually after 10 years, which is the time of starting corrosion.
Moreover, it can be observed that the reliability index in case of $\rho = 4\%$ is higher than that in case of $\rho = 1\%$ until 40 years but after that, the reliability index in case of $\rho = 1\%$ is slightly higher than that in case of $\rho = 4\%$.

It is also noticed that, the rate of decrease in the reliability index of reinforced concrete column after in case of $\rho = 4\%$ is greater than that in case of $\rho = 1\%$.

According to MacGregor\textsuperscript{10}, a value of $\beta = 4.0$ is used in a structural members when consequences of failure become a sever or the failure occurs in brittle manner.

Considering the value of $\beta = 4.0$ (suggested by MacGregor), one can find from Fig. 2 that repair must be done for columns after 17 year in case of $\rho = 1\%$, and after 30 year in case of $\rho = 4\%$.

From Fig. 3, one can find that in case of $\rho = 4\%$ the reliability index is higher than that at $\rho = 1\%$ until 37 years but after that the reliability index in case of $\rho = 1\%$ is higher than that in case of $\rho = 4\%$.

From this figure and by considering $\beta = 4.0$, one can find that the repair must be done after slightly less than 17 years in case of $\rho = 1\%$ and at 20 years in case of $\rho = 4\%$.

In case of corrosion rate $0.114 \text{ mm/year}$ as shown in Fig. 4 the reliability index in case of $4\%$ is higher than that at $\rho = 1\%$ until around 30 year.

Considering $\beta = 4.0$ in Fig. 4, one can find that the repair of the columns must be done after 15 year in case of $\rho = 1\%$ and at 20 year in case of $\rho = 4\%$.

Moreover, one can find that at the begging the probability of failure of the column with $\rho = 4\%$ is lower than that in case of $\rho = 1\%$ until 30, 38, 42 years with corrosion rates $0.064, 0.089, 0.114 \text{ mm/year}$ respectively, then the probability of failure will be higher in case of $\rho = 4\%$.

Therefore, the lower steel ratio is recommended in case of the environmental condition, which increases the rate of corrosion.

**Effect of Initial Time of Corrosion on The Reliability**

It is very important to discuss, also, the case when the steel bars at construction is begin to corrosion. From Fig.5 is discussing this case in two percentages of longitudinal reinforcement $1\%$ and $4\%$.

It can be shown that in Case of $\rho = 4\%$ the reliability index is higher than that in case of $\rho = 1\%$ until 20 year.

In case of $\rho = 4\%$ the rate of decreasing reliability is high.

Initial time of corrosion is the time from constructing until beginning of corrosion.
In this section, three values of initial time of corrosion $T_i$ is considered as shown in Figs. 6, 7.

$T_i=0$ indicates that the outer surfaces of the bars have been already begin to corrode at construction.

At $T_i = 10$ years indicates that corrosion starts after 10 years which represent the normal condition while $T_i= 20$ years indicates that corrosion starts after 20 years.

The third case ($T_i = 20$ years) represents a thick concrete cover which takes time to reach pH less than 5. Whereas the time of beginning corrosion is depending on the thickness of concrete cover, the grade of concrete, and proportional to that density of concrete.

From Figs. 6, 7 one can find that, the reliability of reinforced concrete column increases by increasing the initial time of corrosion. However, the reliability index values are slightly difference between the three initial times after 40 years in case of $\rho = 1 \%$ but in case of $\rho =4 \%$ there is a high reliability range between initial time 20 years and initial time zero.

Therefore, the parameter which increase the initial time is very important to take into consideration in case of $\rho =4 \%$.

**Effect of Eccentricity on The Reliability**

Effect of eccentricity on the reliability index in case of corroded bars with time is presented in Figs. 8, 9 and 10.

In these figures, the reliability index with time are represented in case of $\rho = 1\%$ and $\rho = 4\%$ respectively, with two values of extreme of eccentricity specify by the code EC-89.

Figs. 8, 9 and 10 show the effect of reliability index with time in case of eccentrically loaded column ($e=0.05h$) for different corrosion rates is lower than that for no eccentricity.

From these figures and considering that repair begin when the reliability index reaches to 4.0 (specify by MacGregor). In case of corrosion rate 0.064 mm/year the repair is better to begin at 13 years for $\rho = 1\%$ and 16 years for $\rho = 4\%$.

At rate of corrosion equal to 0.089 mm/year the repair should be begin time in two values of steel is slightly less than the previous rate of corrosion.

However, at rate of corrosion equal to 0.114 mm/year the repair will be at 12 years for $\rho = 1\%$ but at 15 years for $\rho = 4\%$.

From the previous discussion, one can conclude that repair of column with $e = 0.05 \text{ h}$ should be done at least after 12 years and at most 16 years according to the corrosion rate and steel ratio.
On the other hand, the repair of concentrically loaded column should be done at least after 15 years and at most 37 years according to the corrosion rate and steel ratios.

**Conclusions**

The reliability of reinforced concrete column under corrosion attack is discussed taking into consideration two percentages of longitudinal steel bars (ρ =1% and ρ =4%), different corrosion rates, different rates of carbonation which related to different corrosion initial time and the increase of concrete strength with time. Moreover, the effect of corrosion on the reliability index is discussed.

From this study one can conclude the following:

1. The case of ρ = 4% has a higher reliability index than that for case of ρ=1%
2. The repair of concentrically loaded column should be done at least after 15 years and at most 30 years according to the corrosion rate and steel ratio.
3. The reliability index increases by increasing the initial time of corrosion and the initial time has more serious effect in case of ρ = 4%.
4. In general the reliability of RC columns in case of no eccentricity is higher than that in case of eccentricity.
5. The repair of column with e= 0.05 h should be done at least after 12 years and at most 16 years according to the corrosion rate and steel ratio.
6. It is not prefer to use steel bars have a sign of corrosion.

**References**


10. MacGregor J.G.,(1976) “Safety and Limit states Design For Reinforced Concrete”, Published in Canada by The University of alberta
Table 1  Statistical parameters and distributions

<table>
<thead>
<tr>
<th>Design Variables</th>
<th>Specified</th>
<th>Mean in Situ</th>
<th>Mean in situ/Specified</th>
<th>σ</th>
<th>C.O.V</th>
<th>Distribution</th>
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<td>$F_{cu}$ (N/mm²)</td>
<td>19.6</td>
<td>19.3</td>
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<tr>
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<td>L (Kg/m²)</td>
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<td>42.52</td>
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<tr>
<td>D (Kg/m²)</td>
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<tr>
<td>E</td>
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</table>
Fig. 1 Uniform corrosion in a reinforcing bar
[from Frangopol et al (1997)]
Fig. 2 Effect of steel percentage on the reliability index of RC column at corrosion rate $C_r = 0.064$ mm/year

Fig. 3 Effect of steel percentage on the reliability index of RC column at corrosion rate $C_r = 0.089$ mm/year
Fig. 4 Effect of steel percentage on the reliability index of RC column at corrosion rate $C_r = 0.114$ mm/year

Fig. 5 Effect of percentage of steel on reliability index at corrosion initiation time zero
Fig. 6 Effects of corrosion rate on the reliability index at reinforcement ratio $\rho = 1\%$

Fig. 7 Effects of corrosion rate on the reliability index at reinforcement ratio $\rho = 4\%$
Fig. 8 Effect of reinforcement ratio on the reliability index in case of small eccentricity and corrosion rate $C_r = 0.064$ mm/year

Fig. 9 Effect of reinforcement ratio on the reliability index in case of small eccentricity and corrosion rate $C_r = 0.089$ mm/year
Fig. 10 Effect of reinforcement ratio on the reliability index in case of small eccentricity and corrosion rate $C_r = 0.114$ mm/year